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Prediction of Two-Phase Flow Distribution in Parallel Pipes using Stability Analysis

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Two-phase gas liquid flow in pipes is a complex process. One of the problems that is hardly understood is how the two phases are distributed among two or more parallel lines with a common inlet manifold. Steady-state analysis yields multiple steady-state solutions. Linear and nonlinear (simulation) stability analyses are performed in order to determine the actual distribution of the flow that will take place in a real system. The analysis shows that when there are four parallel pipes, for example, the two-phase flow mixture from the common inlet manifold can choose to flow in one, two, three, or in all four pipes, depending on the flow rates of the liquid, and the gas and on the pipes inclination. For low-flow rates of gas and liquid, the flow tends to take place only in one line, while stagnant liquid columns are present in the other three pipes. As the flow rate increases the flow will take place in 2, 3 and finally in 4 pipes. Experimental data confirm the analysis although matching is only approximate.
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Introduction

The behavior of two-phase flow in parallel pipes with a common feed is quite complex and difficult to predict. One of the problems that is hardly understood is how the two phases are distributed among the lines. One may assume that when flow in a single pipe splits into several pipes the liquid and gas ratio in each of the pipes will be the same as in the main line. This is not always the case, and, depending on the geometry and lines diam. there are cases that the flow is strongly preferential, namely most of the gas can enter one pipe, and the liquid into the other pipe. This type of behavior was the subject of considerable attention. Most reported models on splitting in impacting junctions, and side-arms are empirical and, therefore, are applicable for limited flow conditions.[1–11]

Considerable amount of work on flow in parallel pipes were performed for a boiling system where stability and flow-distribution were the main objective.[12–15] Note, however, that for boiling systems the splitting in the inlet manifold is when the fluid is in a single phase. Only after splitting into the parallel pipes, boiling and two-phase flow, takes place.

Taitel et al.[16] investigated experimentally the distribution of gas and liquid in four parallel pipes with a common manifold. They found an interesting phenomenon, that under certain conditions the two phases "choose" to flow only in 1, 2, 3 or 4 pipes (out of 4), while stagnant liquid columns are observed in the other pipes. Their analysis shows that multiple steady-state solutions may satisfy the conditions of equal-pressure drop in the 4 pipes. They predicted the actual solution, namely the number of pipes in which two-phase flow takes place, and the number of liquid columns by using a speculative criterion of minimum pressure drop of the system.[17]

In this work we consider a better alternative, namely a rigorous stability analysis for the determination of the flow distribution in the parallel pipes. It should be stressed that this analysis is not intended to solve the splitting characteristics in the manifold, but rather to predict the number of pipes that have a stagnant-liquid column.

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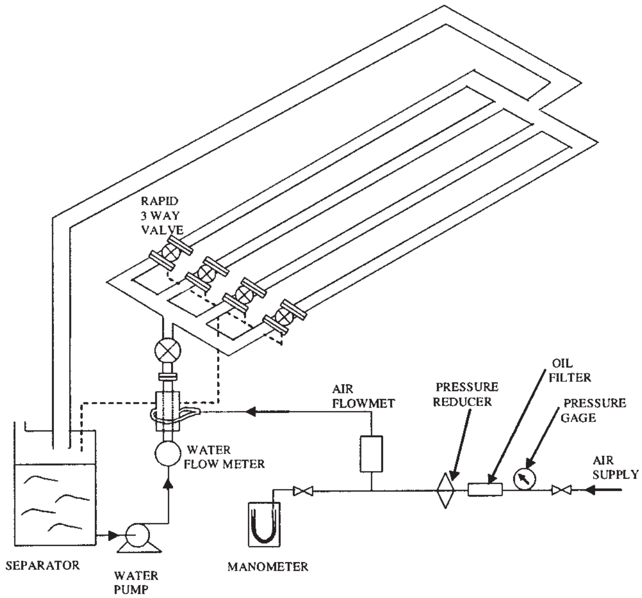

Figure 1. Flow configuration.

Experimental facility

The experimental facility is presented in Figure 1. The system consists of a steel frame supporting a transparent Perspex four parallel pipes, with common inlet and outlet. Each pipe has an internal dia. of 0.026 m and is 6 m long. The frame can be rotated upwards within the range of 0 to 30 degrees.

Water and air are mixed in a specially constructed inlet device before entering the common manifold. All experiments were conducted using air and water at room-temperature and atmospheric outlet pressure.

Air is supplied from a central high-pressure line. A pressure reducer maintains a constant inlet gage of 0.1 MPa. The air flow rate is measured using a turbine flow meter for the high-range of flow rates, and a rotameter for low-flow rates. The water flows in a closed loop, and the flow rates are measured with an electromagnetic-flow meter.

The main experimental results indicate that for the horizontal case the flow always takes place in the four pipes. For inclined pipes various distribution characteristics were observed depending on the liquid and gas flow rates at the inlet to the manifold. For low-flow rates of liquid and gas the gas liquid mixture "prefers" to flow through a single pipe, while stagnant liquid columns partially fill the other three pipes. For increasing liquid and gas flow rates we may see gas and liquid flow in two pipes, while stagnant liquid columns are observed in the other two pipes. For further increase of the flow rates, two phase flow is observed in three pipes, and finally for higher flow rates, the flow takes place in all four pipes.[16]

The main objective of the present work is to predict the flow distribution and compare it with the experimental results.

Analysis

Figure 2 shows such a system. Liquid and gas, in a common vessel, are pressed by a pressure P_0 into two pipes regulated with control valves. The gas and liquid are mixed before entering a common manifold that lead into four (in this case) parallel pipes.

To analyze the stability of this complex system an approximate momentum equation for the two-phase flow mixture in each of the parallel pipes is used as suggested by Akagawa et al.[12]

$$M\frac{dU}{dt} = A(P_{in} - P_{out}) - \tau SL - Mg\sin\beta \qquad (1)$$

where M is the mass of the mixture in each pipe, U is the average mixture velocity, A is the cross sectional area, S is the pipe perimeter, τ the average wall shear stress, L is the pipe length and β is the inclination angle (positive for upward flow).

The shear stress is expressed with the friction factor f

$$\tau = f\frac{\rho U^2}{2} \qquad (2)$$

Substituting Eq. 2 in Eq. 1, and replacing U with the flow rate $Q = UA$ yields,

$$\frac{dQ}{dt} = \frac{A}{\rho L}(P_{in} - P_{out}) - fQ^2\frac{S}{2A^2} - Ag\sin\beta \qquad (3)$$

The flow rate Q is the sum of the liquid flow rate, and the gas flow rate $Q = Q_L + Q_G$, thus,

$$\frac{d(Q_L + Q_G)}{dt} = \frac{A}{\rho L}(P_{in} - P_{out}) - f(Q_L + Q_G)^2\frac{S}{2A^2} - Ag\sin\beta \qquad (4)$$

The average density in the line, ρ is given by

$$\rho = \alpha\rho_G + (1 - \alpha)\rho_L \qquad (5)$$

where the void fraction α, using the drift flux model is given by

$$\alpha = \frac{Q_G}{C(Q_L + Q_G) + AU_d} \qquad (6)$$

In Eq. 6 C is the distribution parameter, about 1.2 for slug flow, and U_d is the drift velocity equal about[18]

$$U_d = 0.54\cos\beta\sqrt{gD} + 0.35\sin\beta\sqrt{gD} \qquad (7)$$

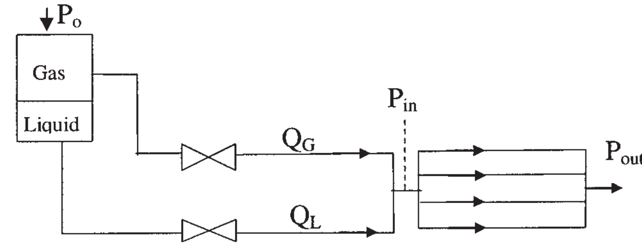

Figure 2. Flow of parallel pipes system.

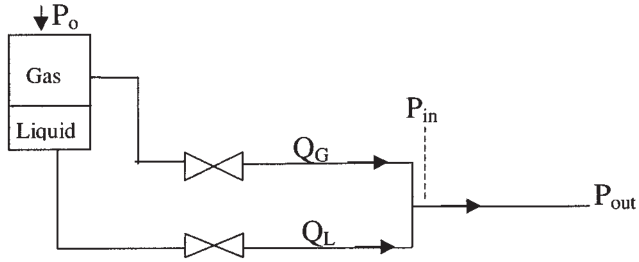

Figure 3. Case of a single pipe.

Case of a single pipe

The simplest case to analyze is when the gas and liquid enter a single pipe as shown in Figure 3. The pressure drop in the line is $P_{in} - P_{out}$. On the other hand the pressure drop in the external supply system is $P_0 - P_{in}$. It is assumed that the pressure drop in the supply lines takes place mainly in the valves, and that the pressure drop is proportional to the flow rate squared. Namely

$$P_0 - P_{in} = N_L Q_L^2 \tag{8}$$

$$P_0 - P_{in} = N_G Q_G^2 \tag{9}$$

Equations 8 and 9 suggest that the ratio of the liquid and gas flow rates is constant, namely

$$Q_L = \sqrt{\frac{N_G}{N_L}}\, Q_G \tag{10}$$

Substituting Eqs. 9 and 10 into 4, and setting the time derivative to zero, yields the steady-state solutions for the gas flow rate and the inlet pressure

$$P_{in} = P_{out} + f\rho(Q_G\sqrt{N_G/N_L} + Q_G)^2 \frac{SL}{2A^3} + \rho Lg \sin\beta = P_0 - N_G Q_G^2 \tag{11}$$

Equation 11 is solved for the flow rates under steady-state conditions. The solutions are obtained when $P_{in,external}$ of the supply system is equal to $P_{in,internal}$ of the "internal" line (see Figure 4). The solid line shows a typical pressure drop ($P_{in,internal} - P_{out}$) for an upward inclined line as a function of the gas flow rate, while the dashed line shows the external supply system behavior ($P_{in,external} - P_{out}$). As shown, we may have two steady-state solutions A and B.

Stability analysis

A linear stability analysis is performed on the steady-state solutions. Equation 4 is rewritten in the form

$$P_{in} = P_{out} + F(Q_L + Q_G) + m\frac{d(Q_L + Q_G)}{dt} \tag{12}$$

where $m = M/A^2$, and F is given by

$$F = f(Q_L + Q_G)^2 \frac{\rho LS}{2A^3} + \rho Lg \sin\beta = F(Q_L, Q_G) = F(Q_G\sqrt{N_G/N_L}, Q_G) = F(Q_G) \tag{13}$$

F is a function of the liquid and gas flow rates. However, since the liquid flow rate is related to the gas flow rate (see Eq. 10) then F can be considered as a function of Q_G only.

Using Eq. 12 and introducing a small perturbation in the gas flow rate q_G around the steady-state solution $Q_{G,0}$, we obtain the following equation for the perturb variable q_G

$$\frac{dP_{in}}{dQ_G} q_G = \frac{dF}{dQ_G} q_G + m(\sqrt{N_G/N_L} + 1)\frac{dq_G}{dt} \tag{14}$$

Substituting $\delta_G e^{\lambda t}$ for q_G, and solving for the eigenvalue λ yields

$$\lambda = \frac{\dfrac{dP_{in}}{dQ_G} - \dfrac{dF}{dQ_G}}{m(\sqrt{N_G/N_L} + 1)} \tag{15}$$

The conditions for stability is $\lambda \leq 0$ that is

$$\frac{dP_{in}}{dQ_G} \leq \frac{dF}{dQ_G} \tag{16}$$

This result shows that solution A is unstable, and solution B is stable. Note that this result can be also obtained intuitively by considering small perturbation on the gas flow rate. When small positive perturbation is applied to state A then the external pressure exceeds the internal pressure in the pipe causing a further increase of the flow rate. Using the same logic, solution B is shown to be stable. This type of instability is commonly termed as Ledinegg instability since it was mentioned first by Ledinegg.[19]

Transient simulation—single pipe

Equation 4 is used for transient simulations that allow the solution of Q_G (and Q_L) as a function of time

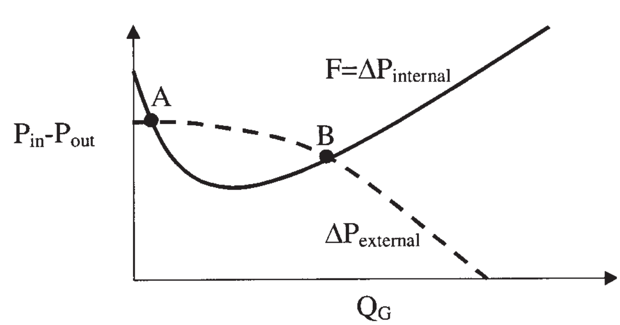

Figure 4. Steady-state solutions for a single pipe flow.

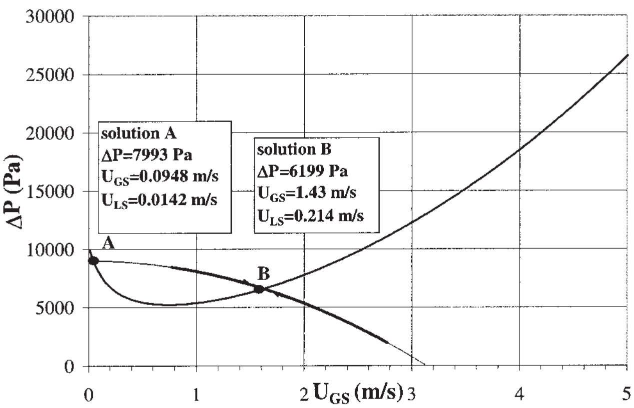

Figure 5. Transient simulation path, P_0 = 0.1080 MPa, P_{out} = 0.1 MPa, D = 2.6 cm, L = 6 m, μ_L = 0.001 Kg/m s, μ_G = 0.0000186 Kg/m s, β = 10^0, f = 0.005, N_G = 3.25 × 10^9 Pa/(m^3/s)^2, N_L = 1.45 × 10^{11} Pa/(m^3/s)^2.

$$(1 + \sqrt{N_G/N_L})\,\frac{dQ_G}{dt} = \frac{A}{\rho L}\,(P_0 - N_G Q_G^2 - P_{out}) - fQ_G^2(1 + \sqrt{N_G/N_L})^2\,\frac{S}{2A^2} - Ag\sin\beta \quad (17)$$

Note that the density is also a function of the gas flow rate (see Eqs. 5 and 6).

Figure 5 shows an example of the path of the transient trajectory responding to a finite disturbance on Q_G. The trajectory is shown on a plane of $\Delta P = P_{in} - P_{out}$ vs. U_{GS} using properties as in our experimental tests. "A" is a steady-unstable solution, and "B" is a steady-stable solution. As can be seen starting at a state right or left to the steady-state stable solution B will end up at solution B. Likewise, the trajectory will end up at point B if we start slightly to right of point A. If we start to the left of point A we will end up with stagnant liquid because the external pressure P_0 is unable to overcome the hydrostatic pressure generated by the liquid column.

Case of two parallel pipes

The steady-state force balances for each of the two parallel pipes are (see Eq. 11)

$$P_{in} = P_{out} + f_1\rho_1(Q_{L,1} + Q_{G,1})^2\,\frac{SL}{2A^3} + \rho_1 Lg\sin\beta = P_0 - N_G Q_G^2 \quad (18)$$

$$P_{in} = P_{out} + f_2\rho_2(Q_{L,2} + Q_{G,2})^2\,\frac{SL}{2A^3} + \rho_2 Lg\sin\beta = P_0 - N_G Q_G^2 \quad (19)$$

Note that $Q_{L,2} = Q_L - Q_{L,1} = Q_G\sqrt{(N_G/N_L)} - Q_{L1}$ and $Q_{G,2} = Q_G - Q_{G,1}$.

We have 2 equations and 3 unknowns: Q_G, $Q_{L,1}$ and $Q_{G,1}$. Thus, infinite number of solutions will satisfy these equations. A procedure to obtain the solutions is to consider $Q_{G,1}$ as an input and solve for $Q_{L,1}$ and Q_G. In Figure 6 the liquid flow rate ratio in one pipe is plotted against the gas flow rate ratio in the same pipe. The curved line shows the collection of all possible steady state solutions that satisfy equal pressure drop in the two parallel pipes.

An additional restriction is the splitting characteristics at the inlet to the manifold. In order to simplify the solution, and to avoid an enormous complexity we used a simple assumption for the splitting characteristics, namely that the ratio of the gas to liquid in each of the parallel pipes is the same as in the main line entering the manifold. Namely $Q_L/Q_G = Q_{L,1}/Q_{G,1} = \sqrt{N_G/N_L} \equiv R_1$. The 45° straight line represent these characteristics. The intersection of both curves yields three steady state solutions A, B and C. Note that solution B is a trivial solution for the case where the mixture flow rate in both pipes is the same.

Stability analysis—two pipes

Once steady-state solutions are obtained the stability of the solutions is analyzed using Eq. 14 for each of the pipes, namely

$$\frac{dP_{in}}{dQ_G}\,(q_{G,1} + q_{G,2}) - \frac{dF_1}{dQ_{G,1}}\,q_{G,1} - m_1(R_1 + 1)\,\frac{dq_{G,1}}{dt} = 0 \quad (20)$$

$$\frac{dP_{in}}{dQ_G}\,(q_{G,1} + q_{G,2}) - \frac{dF_2}{dQ_{G,2}}\,q_{G,2} - m_2(R_2 + 1)\,\frac{dq_{G,2}}{dt} = 0 \quad (21)$$

where

$$q_G = q_{G,1} + q_{G,2} \quad R_1 = R_2 = \sqrt{N_G/N_L} \quad (22)$$

Substituting $\delta_{G,1}e^{\lambda t}$ for $q_{G,1}$ and $\delta_{G,2}e^{\lambda t}$ for $q_{G,2}$ yields

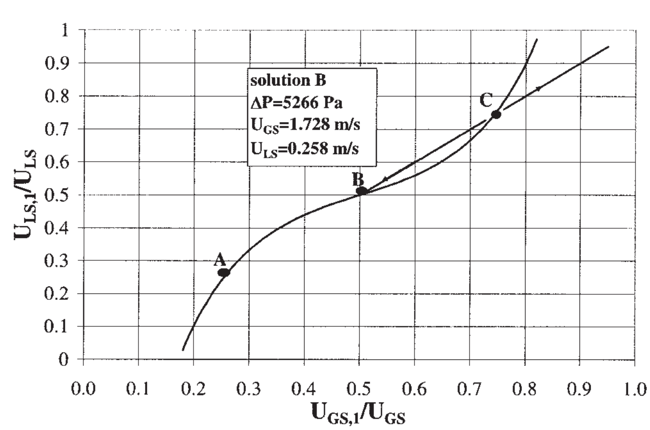

Figure 6. Case of two pipes—1st example, P_0 = 0.1080 MPa, P_{out} = 0.1 MPa, D = 2.6 cm, L = 6 m, μ_L = 0.001 Kg/m s, μ_G = 0.0000186 Kg/m s, β = 10^0, f = 0.005, N_G = 3.25 × 10^9 Pa/(m^3/s)^2, N_L = 1.45 × 10^{11} Pa/(m^3/s)^2.

$$\frac{dP_{in}}{dQ_G}(\delta_{G,1}+\delta_{G,2})-\frac{dF_1}{dQ_{G,1}}\delta_{G,1}-m_1(R_1+1)\lambda\delta_{G,1}=0$$

$$\frac{dP_{in}}{dQ_G}(\delta_{G,1}+\delta_{G,2})-\frac{dF_2}{dQ_{G,2}}\delta_{G,2}-m_2(R_2+1)\lambda\delta_{G,2}=0 \quad (23)$$

The system is stable for λ equal or less than 0. The eigenvalue λ is a solution of the determinant

$$\begin{vmatrix} \frac{dP_{in}}{dQ_G}-\frac{dF_1}{dQ_{G,1}}-m_1(1+R_1)\lambda & \frac{dP_{in}}{dQ_G} \\ \frac{dP_{in}}{dQ_G} & \frac{dP_{in}}{dQ_G}-\frac{dF_2}{dQ_{G,2}}-m_2(1+R_2)\lambda \end{vmatrix}=0 \quad (24)$$

Note that by using relation 10, P_{in}, F_1 and F_2 are functions only of the gas-flow rate. Equation 24 is a quadratic equation for λ resulting in two values. Considering constant input of gas and liquid flow rates $dP_{in}/dQ_G \rightarrow -\infty$. The eigenvalues λ_1 & λ_2 were solved numerically for the steady states A, B and C. It is obtained that only solution B is stable.

Another example is given in Figure 7. The conditions here are the same as in Figure 6 with the exception that the external pressure P_0 is slightly reduced. In this case we have only one steady-state solution (point B). Calculation of the eigenvalues λ shows that B is an unstable solution.

The physical interpretation of the aforementioned stability analysis is as follows: for the case of multiple steady state solutions only the one that is stable will exist, and the system will accommodate two-phase flow in both pipes. When there is no stable solution it means that flow in both pipe is not possible, namely the flow will take place only in one pipe, and a stagnant liquid column will take place in the other pipe.

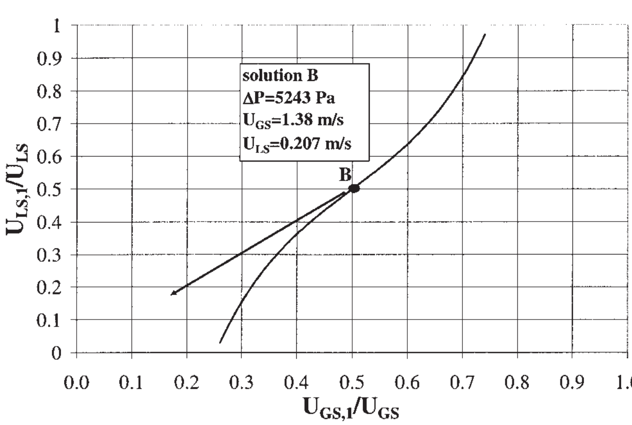

Figure 7. Case of two pipes—2nd example, P_0 = 0.1070 MPa, P_{out} = 0.1 MPa, D = 2.6 cm, L = 6 m, μ_L = 0.001 Kg/m s, μ_G = 0.0000186 Kg/m s, β = 10^0, f = 0.005, N_G = 3.25 × 10^9 Pa/(m^3/s)^2, N_L = 1.45 × 10^{11} Pa/(m^3/s)^2.

Transient simulation—2 pipes

Two transient momentum equations are used (see Eq. 4)

$$\frac{dQ_{G,1}}{dt}+\frac{dQ_{L,1}}{dt}=\frac{A}{\rho_1 L}(P_0-N_G Q_G^2-P_{out})-f_1(Q_{G,1}+Q_{L,1})^2\frac{S}{2A^2}-Ag\sin\beta \quad (25)$$

$$\frac{dQ_{G,2}}{dt}+\frac{dQ_{L,2}}{dt}=\frac{A}{\rho_2 L}(P_0-N_G Q_G^2-P_{out})-f_2(Q_{G,2}+Q_{L,2})^2\frac{S}{2A^2}-Ag\sin\beta \quad (26)$$

Adding these two equations yields

$$\frac{dQ_G}{dt}+\frac{dQ_L}{dt}=\frac{A}{\rho_1 L}(P_0-N_G Q_G^2-P_{out})-f_1(Q_{G,1}+Q_{L,1})^2\frac{S}{2A^2}-Ag\sin\beta+\frac{A}{\rho_2 L}(P_0-N_G Q_G^2-P_{out})-f_2(Q_{G,2}+Q_{L,2})^2\frac{S}{2A^2}-Ag\sin\beta \quad (27)$$

Using $Q_L = Q_G\sqrt{N_G/N_L}$, and $Q_{L,1} = Q_{G,1}\sqrt{N_G/N_L}$ yields the following two differential equations for $Q_{G,1}$ and Q_G.

$$(1+\sqrt{N_G/N_L})\frac{dQ_{G,1}}{dt}=\frac{A}{\rho_1 L}(P_0-N_G Q_G^2-P_{out})-f_1(Q_{G,1}+Q_{L,1})^2\frac{S}{2A^2}-Ag\sin\beta \quad (28)$$

$$(1+\sqrt{N_G/N_L})\frac{dQ_G}{dt}=\frac{A}{\rho_1 L}(P_0-N_G Q_G^2-P_{out})-f_1(Q_{G,1}+Q_{L,1})^2\frac{S}{2A^2}-Ag\sin\beta+\frac{A}{\rho_2 L}(P_0-N_G Q_G^2-P_{out})-f_2(Q_{G,2}+Q_{L,2})^2\frac{S}{2A^2}-Ag\sin\beta \quad (29)$$

Note that ρ_1, ρ_2 and α_1, α_2 are given by Eqs. 5 and 6.

The results of the simulation are shown by the paths in Figures 6 and 7. As expected the path converge into the steady-state solution (B in Figure 6), and run away from the unstable steady-state solutions (A and C in Figure 6, and B in Figure 7).

General case of n number of parallel pipes

Having n pipes in parallel, the steady state momentum equation for each of the pipes is

$$P_{in}=P_{out}+f_i\rho_i(Q_{L,i}+Q_{G,i})^2\frac{SL}{2A^3}+\rho_i Lg\sin\beta=P_0-N_G Q_G^2 \quad i=1\cdots n \quad (30)$$

The unknowns are Q_G (which also determines Q_L), $Q_{G,i}$ and $Q_{L,i}$. Thus, we have n equations with $2(n-1)+1$ unknowns Q_G, $Q_{G,1}$, $Q_{L,1}$, $Q_{G,2}$, $Q_{L,2} \cdots Q_{G,n-1}$, $Q_{L,n-1}$. For a given set of $Q_{G,,i}$, such as system can be solved for Q_G and $Q_{L,i}$. In this case one obtains infinite steady-state solutions. In order to obtain a finite number of solutions it is further assumed, as before, that the ratio of gas to liquid in each of the parallel pipes is equal to this ratio in the main entrance line to the manifold.

Stability analysis—multiparallel pipes

Once steady-state solutions are obtained, the stability of the solutions is analyzed using Eq. 14 for each of the pipes, namely

$$\frac{dP_{in}}{dQ_G}(q_{G,i}+\cdots+q_{G,n})-\frac{dF}{dQ_{G,i}}q_{G,i}-m_i(R_i+1)\frac{dq_{G,i}}{dt}=0 \tag{31}$$

Substituting $\delta_{G,i}e^{\lambda t}$ for $q_{G,i}$ yields

$$\frac{dP_{in}}{dQ_G}(\delta_{G,i}+\cdots+\delta_{G,n})-\frac{dF}{dQ_{G,i}}\delta_{G,i}-m_i(R_i+1)\lambda\delta_{G,i}=0 \quad i=1\cdots n \tag{32}$$

The solution of the steady states, and the calculation of the corresponding eigenvalues λ may be quite cumbersome. Based on our previous analysis we conclude that when we have multiple steady-state solutions the only solution that may be stable is the one of equal distribution of the mixture among the pipes. However, when the even distribution solution is unstable there is no stable solution for the flow in n pipes. Thus, the stability analysis is performed only for the case of equal distribution of the two-phase flow mixture. The solution for the eigenvalues λ's in this case are

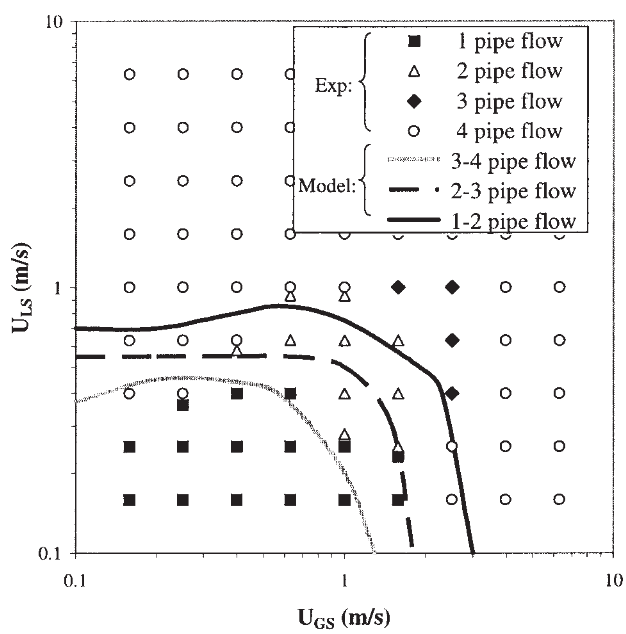

Figure 8. Flow splitting in 4 pipe system, *D* = 2.6 cm, β = 5°.

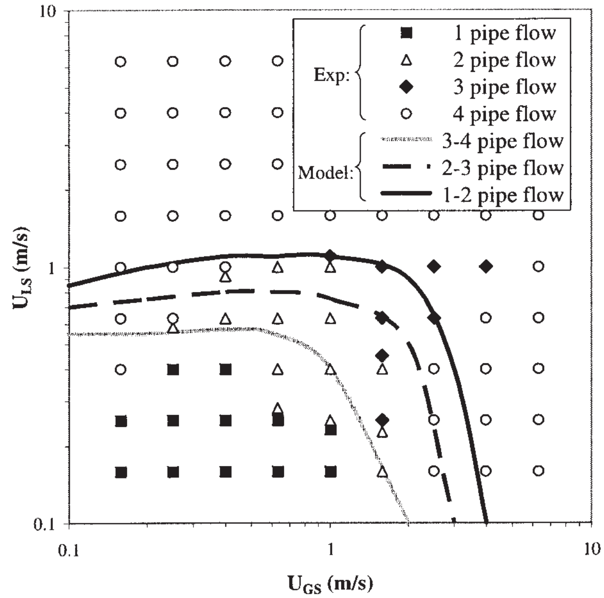

Figure 9. Flow splitting in 4 pipe system, *D* = 2.6 cm, β = 10°.

$$\lambda_1=\frac{n\dfrac{dP_{in}}{dQ_G}-\dfrac{dF_i}{dQ_{G,i}}}{m_i(R_i+1)}$$

$$\lambda_i=\frac{-\dfrac{dF_i}{dQ_{G,i}}}{m_i(R_i+1)} \quad i=2\cdots n \tag{33}$$

The condition of stability is satisfied when all eigenvalues are less or equal zero

$$n\frac{dP_{in}}{dQ_G}-\frac{dF_i}{dQ_{G,i}}\le 0$$

$$-\frac{dF_i}{dQ_{G,i}}\le 0 \quad i=2\cdots n \tag{34}$$

For the case where the inlet gas (and liquid) flow rates are constant, dP_{in}/dQ_G approaches $-\infty$, the first stability condition is always satisfied. Thus, only the condition $-dF_i/dQ_{G,i}\le 0$ at the steady-state solutions has to be satisfied. In practice this situation of constant input flow rates takes place, when constant displacement pumps and compressors are used or when the external pressure P_0, and the resistances of the valves are very high.

Results and Discussion

As previously explained, criterion (Eq. 34) is applied to the steady-state case of even distribution of the two phases among the parallel pipes. Consider the case of 4 parallel pipes with common manifold as used in our experiment. When Eq. 34 is satisfied for the solution of even distribution in four pipes it means that the flow is stable, and will actually exist in the four

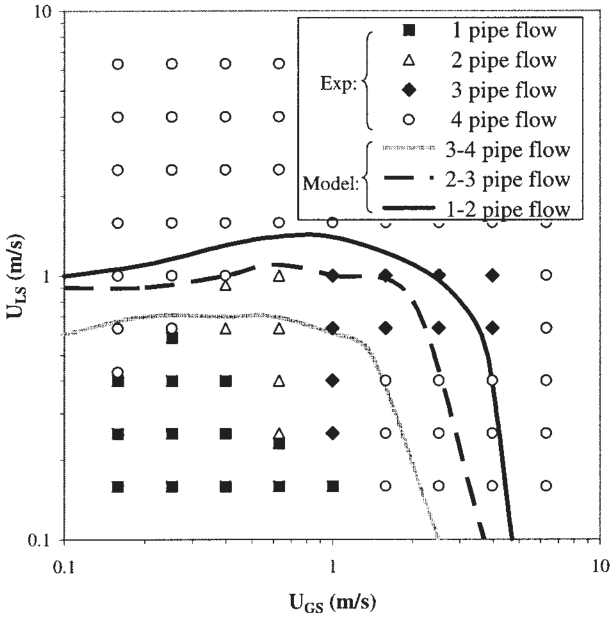

Figure 10. Flow splitting in 4 pipe system, D = 2.6 cm, $\beta = 15^{o}$.

pipes. If not, stability is checked for flow in three pipes. If it is stable for three pipes then the two-phase flow will take place in three pipes and a stagnant liquid column will be formed in the fourth pipe. If the flow in three pipes is not stable the system is checked for two pipes, and so on.

Figures 8, 9 and 10 show the results of the stability analysis, as well as our experimental results for 5, 10 and 15 degrees of pipe inclination, respectively. For low-flow rates of gas and liquid the flow takes place only in one pipe while stagnant liquids columns are present in the other three pipes. As the flow rates of the liquid and the gas increase the flow takes place in two pipes, then in three pipes, and finally in all four pipes. Stability analysis shows that for low-flow rate the flow is unstable in four, three, and two pipes. As the flow rate of liquid and gas increases, the flow becomes stable in two, three and finally in four pipes. Thus, for high-flow rates the flow takes place in all four pipes. The lines demarcate the regions of 1, 2, 3 and 4 pipe flow regions, based on the stability analysis. As can be seen the theory predict the experimental trend fairly well.

In our previous article,[17] the theoretical results for the same phenomena were based on the idea that the system will take the configuration of minimum pressure drop. The results based on the minimum pressure drop speculation yielded fairly reasonable results, yet the results, based on the present stability analysis fit much better the experimental results.

Summary and Conclusions

Flow distribution of gas and liquid in four parallel pipes is investigated. The analysis is simplified by the assumption that the splitting characteristic is such that the liquid to gas ratio is kept constant. The solution yields multiple steady state solutions for the flow distribution. Stability analysis and dynamic simulations are performed in order to determine the actual flow distribution.

Results were calculated for upwards inclination of 0, 5, 10 and 15 degrees. For the horizontal case the flow takes place in all four pipes. For the inclined pipes various flow configuration are obtained. For low-liquid and gas flow rates the two-phase mixture "prefers" to flow in a single pipe, while stagnant liquid columns fill part of the other three pipes. As the flow rates of liquid and gas increase, flow in two, three and eventually in four pipes takes place.

The theoretical results compare fairly well with our experimental data taken in a four parallel pipe system, 6 m long, and 2.6 cm dia.

Notation

A = cross sectional area
C = distribution parameter
D = pipe diameter
f = friction factor
F = friction function
g = gravitational constant
L = length
m = M/A^2
M = mass
n = number of pipes
N = pressure-loss coefficient
P = pressure
q = perturb volumetric flow rate
Q = volumetric flow rate
R = flow rate ratio, Q_L/Q_G
S = periphery
t = time
U = velocity

Greek letters

α = void fraction
β = angle of inclination (positive for up flow)
δ = perturb amplitude
λ = eigenvalue
ρ = density
τ = shear stress

Subscripts

0 = extermal
d = drift
G = gas
in = at the inlet manifold
L = liquid
out = at outlet

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